Educação: entre teoria e prática

Volume III

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Organizadores

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Apresentação

O e-book "Educação entre Teoria e Prática - Volume III" apresenta uma coletânea de artigos que exploram as interfaces entre teoria e prática na educação contemporânea. A obra oferece uma visão abrangente dos desafios e oportunidades que moldam a educação, desde a educação a distância e a inteligência artificial até o multilateralismo e a formação de professores.

Os capítulos iniciais mergulham no universo da Educação a Distância (EaD), analisando os impactos da pandemia de COVID-19 e a crescente importância da inteligência artificial como ferramenta para personalizar o aprendizado. A obra também aborda a dimensão global da educação, discutindo o papel do multilateralismo na construção de um futuro mais justo e equitativo.

A formação de professores é outro tema central. Os capítulos dedicados a essa temática exploram as potencialidades das tecnologias digitais para a formação continuada de professores, bem como as implicações da história da matemática para o ensino de geometria. Estudos de caso demonstram como o binômio teoria e prática se revela em diferentes áreas do conhecimento, como a Educação Física e a Matemática.

A Base Nacional Comum Curricular (BNCC) e a educação profissional são analisadas sob a perspectiva da prática e do desenvolvimento de competências. Um estudo de caso ilustra como uma escola pode implementar uma proposta pedagógica inovadora que integra a teoria e a prática.

O e-book também apresenta um levantamento sobre a formação continuada de professores no estado do Amazonas, evidenciando a importância das políticas públicas para garantir a qualificação dos profissionais da educação.

Este e-book é destinado a professores, pesquisadores, gestores educacionais, estudantes de graduação e pós-graduação em educação, e a todos aqueles que se interessam pelas novas tendências e desafios da educação. A obra contribui para o debate sobre as questões mais relevantes da educação contemporânea, oferecendo subsídios para a prática docente, a formulação de políticas públicas e o desenvolvimento de pesquisas na área.

Os organizadores

Sumário

Chapter 3

Teacher's mathematical work in quadrilateral teaching using digital technology and the history of mathematics

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INTRODUCTION

Digital technology offers teachers in continuous training the opportunity to link aspects of the history of mathematics to their knowledge of mathematics, and even how to make or reorganize work plans, among others. This work, which is part of a qualitative research project (Hernández-Sampieri; Fernández; Baptista, 2014), associates teacher training in the mastery of geometry and the use of digital technology — seeking to connect teachers' technological knowledge with the support of the Technological Pedagogical Content Knowledge (TPACK) model and their *idoine* Mathematical Working Space (MWS) — with the use of the history of mathematics for teaching.

According to Drijvers, Ball, Barzel, Heid, Cao and Maschietto (2016), the complex relation between technology, mathematics and education is not only due to it in itself, but it has existed since different "artifacts" are used for teaching mathematics. There are technologies with different competence levels or degrees depending on the what they were created for and how they are used in mathematics teaching. This is where the use of the history of mathematics becomes really important.

Regarding digital technology, Salazar, Gaona & Richard (2022) state, from an MWS position, that digital artifacts for mathematics teaching and learning are a set of propositions characterized by being executable by an electronic machine with historical intelligence and relative epistemological validity. In addition, Salazar, García-Cuéllar, Vivas & Peñaloza (2023) state that the use of digital technology allows different representations of mathematical objects to be made which, when explored, promote understanding. Likewise, the authors note that the knowledge or experience teachers have using one or another digital technology has an influence on how a given mathematical content is taught, but also on the potential and limitations of the artifact, as well as the needs of the individuals (teachers or students) who will use it (p. 429).

Regarding geometry, Kuzniak and Nechache (2021) investigate how in-service teachers approach mathematical tasks, specifically in geometry. Their study focused on French teachers beginning mas-ter's programs who struggled with geometry despite theoretical knowledge, revealing deficiencies in theorem and property application.

The authors categorized teachers' work based on complete answerless and mathematical accuracy, identifying five distinct approaches: dissector, inspector, explorer, constructor, and calculator. They developed a robust methodology and highlighted the importance of considering school-based paradigms to understand teacher performance.

Regarding teacher training and the integration of digital technologies, there are research groups in the mathematics education community with a growing number of research papers and publications (Salazar & Théry, 2023; Thurm, 2018; Psycharis & Kalogeria, 2017; Pierce & Stacey, 2013). For example, Bowers and Stephens' (2011) study presents different uses of various technologies that teacher trainers could include in their classes. For them, the core of technology integration in teacher training is to stimulate the use of available tools to explain mathematical relations, etc.

Codina and Romero (2016) examined how incorporating digital technology transformed teacher and student roles. They found that students construct meaning within their individual mathematical workspace, influenced by both teacher actions and technology. Additionally, García-Cuéllar and Salazar (2019) highlight digital technology's ability to reveal students' underlying mathematical thinking as evidenced in their problem-solving actions.

Similarly, Kuzniak, Nechache & Salazar (2020) conducted a study with mathematics teachers in continuous training in France and Peru in order to identify their mathematical work in mastering geometry. Likewise, the study seeks to promote the interaction of teachers with digital technology so that they benefit from it according to their use preferences. The main results were that, when mathematics teachers interact with digital technology, they have more difficulties using it than with their knowledge of geometry. They also state that digital technology has a transforming effect on the mathematical work of teachers. However, it is necessary to know its limitations and potentials.

In this regard, Artigue (2011) emphasizes that the effective integration of digital technology in Mathematics Education requires a balance between its epistemic and pragmatic value, avoiding simplistic adaptations of traditional practices and taking advantage of its potential to develop critical thinking and mathematical exploration.

Given these considerations, we believe that teacher training is crucial for establishing connections between digital technology and the history of mathematics in teaching. Specifically, mathematics teacher training, supported by appropriate educational policies, is essential. To foster the mathematical work of students and teachers, we must design tasks that integrate the history of mathematics with digital technology. These tasks should leverage teachers' technological, pedagogical, and mathematical expertise (Salazar & Théry, 2023).

Thus, the question is how to design tasks for teaching and digital technology using the history of mathematics, and which theories of the didactics of mathematics can help us understand this connection.

These are questions that might not be answered in full; however, they will serve as a guide for the connections that we intend to establish.

MATERIAL AND METHODS

Some theoretical tools and their connections

The Mathematical Working Space (MWS) theory is a potential framework for addressing our research questions. This theory emphasizes the individual's role in constructing mathematical understanding through their engagement with mathematical tasks. However, this process is gradual, interactive and complex because the evolution of mathematical knowledge will depend, to a large extent, on the proposed tasks and mathematical actions done to solve them (Kuzniak, Tanguay & Elia, 2016). Regarding the task, this is defined as any type of mathematical exercise, question or problem with clear questions that students are able to solve. In addition, depending on the institution, the mathematical work can be typified: "the mathematics considered by the institution, described in the reference MWS, is developed by the teacher until reaching an *idoine* MWS that allows an effective establishment in class, where each student works on his/her personal MWS" (Gómez-Chacón, Kuzniak & Vivier, 2016).

On the other hand, Kuzniak, Montoya-Delgadillo & Richard (2022) state that the *idoine* MWS allows analyzing how individuals plan, perfect and evolve their mathematical work in a specific educational context. Also, the authors point out that paradigms characterize the mathematical work of the individuals. Thus, in the MWS, a paradigm is a set of beliefs, techniques, and values shared by a scientific group; mathematical mastery is determined by the nature of the objects studied and the paradigms that characterize it, e.g., mastery of geometry, algebra, arithmetic, analysis, etc.

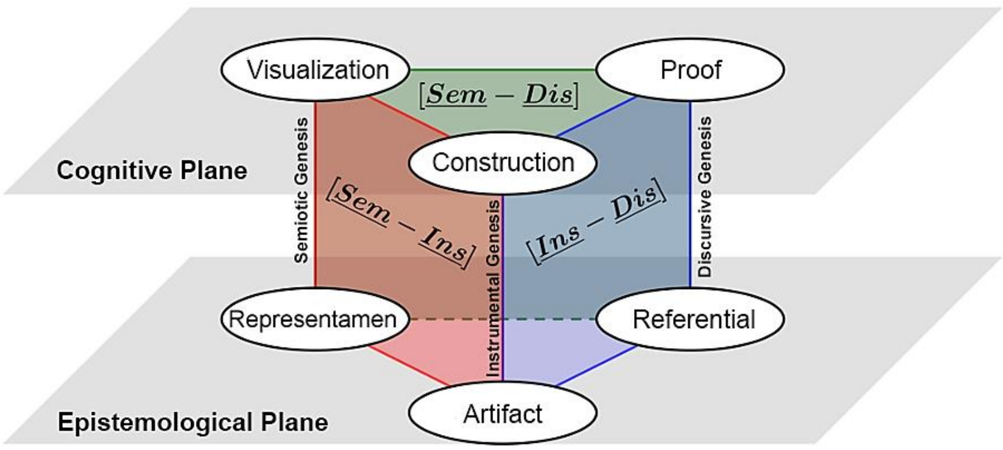

Figure 1. MWS model. Source: Adapted by the authors from Kuzniak et al. (2015, p. 248).

As shown in Figure 1, the epistemological and cognitive planes are articulated in the MWS through the geneses evidenced by the mathematical work. The semiotic genesis is the process associated with signs and representamen (or signifiers); the instrumental genesis allows artifacts to be made operational through construction processes that help achieve the mathematical work; and, the discursive genesis applies the properties of the theoretical reference system to use them in favor of mathematical reasoning and to do not only an iconic validation, but also a graphic or instrumental one. Similarly, Kuzniak & Richard (2014) identify three vertical planes, each of which is defined by the interaction of two geneses: semiotic and instrumental [Sem-Ins], instrumental and discursive [Ins-Dis], and semiotic and discursive [Sem-Dis].

As for the planes, [Sem-Ins] is associated with a semiotic genesis and instrumental genesis, where there are two ways of working, one to construct results (figures, graphs), and another one to interpret the data proposed by the artifacts; [Ins-Dis] is associated to a discursive genesis of the proof and to the instrumental genesis; and [Sem-Dis] is associated to the semiotic and discursive genesis, in which argumentative reasoning is perceived. On the other hand, the mathematical work is characterized by its respective paradigms (depending on the mathematical expertise).

Likewise, Koehler and Mishra's (2015) Technological Pedagogical Content Knowledge (TPACK) model describe teachers' knowledge to teach specific content by using or interacting with technologies. This model has components that explain teachers' TPACK, such as their conceptions about teaching content with technologies; students' learning with technologies; knowledge of the curriculum and other curricular documents; knowledge of teaching and didactic representations; among others.

Furthermore, to describe teachers' knowledge transformation in regards to teaching with technologies, five levels are proposed to include teaching with technologies: to recognize, accept, adapt, explore, and expand. These levels of technology integration proposed by Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, Özgün-Koca, & Kersaint (2009) are explored by the TPACK components to explain the development of the knowledge, skills, and provisions that comprise teachers' knowledge.

On the other hand, for Mendes (2023), implementing a mathematics teaching approach focused on connections involving the history of mathematics associated with technological support through digital media requires a teacher to first use the existence of three masteries involving the history of mathematics as a guide: (a) mastery of knowledge about the historical development of mathematical contents; (b) mastery of knowledge about the development of digital technologies; (c) mastery of theoretical-practical knowledge of the applications available for access, according to the objectives of the user (Mendes, 2023, p.17-19). A simpler and more familiar example of these relations involves the use of these calculator applications to address historical mathematical topics, such as the values of π, or irrational numbers in general. Other mathematization processes of historical mathematical topics in virtual connections to teach mathematics today are exemplified in developments related to the use of other dynamics, such as GeoGebra®, to show exercises that express possibilities of inserting historical topics or historical processes of mathematical production in conjunction with digital media (Mendes, 2023, p. 19).

To effectively create and implement tasks that integrate history and technology, Mendes (2024, p. 18) proposes that teachers delve into historiographic research on mathematics within primary, secondary, and tertiary texts to gain a comprehensive grasp of the subject's historical-conceptual evolution. Only by appropriating this knowledge — explored from the widest possible sources — will it be possible to organize historical sequences that contemplate the conceptual development foreseen in the teaching programs and textbooks in each school year. In this sense, Mendes (2020) states that, in practice, teachers will be able to organize sequential historical movements (SHM) related to historical issues, such as irrationals; quadrature of curves and their conceptual movements; Cavalieri's indivisibles; the concept of variables, functions and differential and integral calculus; complex numbers; analytical geometry before and after the studies of René Descartes; Isaac Newton and Colin Maclaurin's fluxion method; trigonometry of strings, semi-strings, plane and spherical triangles; among other mathematical imaginations or creations.

Proposed approach

As explained above, this qualitative study (Hernández-Sampieri; Fernández; Baptista, 2014) is part of a research project entitled "Mathematical working space of high school teachers and their technological, pedagogical and mathematical content knowledge of geometry", in which participants were in-service high school teachers teaching Mathematics in the first, second or third year of high school in private Peruvian institutions (teaching Peruvian students of 12, 13 and 14 years of age, respectively).

We consider that the theoretical approaches provided by MWS, TPACK, and the history of mathematics approach to teaching drive teachers' mathematical work on a given mathematical content, which in our case is quadrilaterals. As an example, the task "Tensors" based on the investigations of Théry (2023) and Salazar & Théry (2023) is presented, which aims to develop teachers' technological knowledge by using the history of mathematics, i.e., for teachers to show greater TPACK to integrate the history of mathematics and technology in their *idoine* MWS. The task seeks to show how teachers could teach quadrilaterals making use of history and, in the sense of MWS, digital artifacts that allow visualizing and manipulating representations of quadrilaterals, that is, promoting the activation of the semiotic, discursive and instrumental geneses, and, consequently, the activation of the [Sem-Dis] and [Sem-Ins] planes.

To better understand the task, its historical context is explained next: around 3000 B.C., ancient Egyptians used practical geometry with quadrilaterals, for example, to measure land boundaries after the annual flooding of the Nile River and in the construction of pyramids. Egyptian string thinkers divided land by measuring plots after the floods of the Nile made boundaries disappear. They did these using ropes, stakes, and measuring instruments to construct quadrilaterals through right triangles with sacred numbers 3, 4 and 5 Théry (2023; as cited in Sanchez, 2012).

An episode narrated by Proclus reflects the importance Egyptians placed on geometry and, in particular, on the area and perimeter of quadrilaterals. Proclus describes that participants in land division would sometimes confuse their fellows in land distribution by measuring, at their convenience, the longest side of the boundaries between plots. After acquiring a piece of land with more periphery or distance from its lateral neighbors (rectangular in shape), they would then exchange it for a piece of land with a smaller boundary (similar to the shape of a square), and thus getting more land than their neighbors. In addition, by doing so, they gained a reputation for great honesty (Heath, 1981).

This story reflects the knowledge that Egyptians began to develop on the area and perimeter of quadrilaterals, such as the rectangle. In addition, these measurements were also used to calculate taxes owed to the pharaohs, which was based on the perimeter of the land. This story shows that some Egyptians took advantage of the belief that, if a quadrilateral has a larger perimeter, it will also have a larger area. It also shows that, in the time of Proclus, they began to have notions of isoperimetry and optimization. Isoperimetry refers to the relations between figures with equal perimeter. It is also known that they identified the square as the quadrilateral with the largest area among isoperimetric quadrilaterals, according to Théry (2023; as cited in Sánchez, 2012).

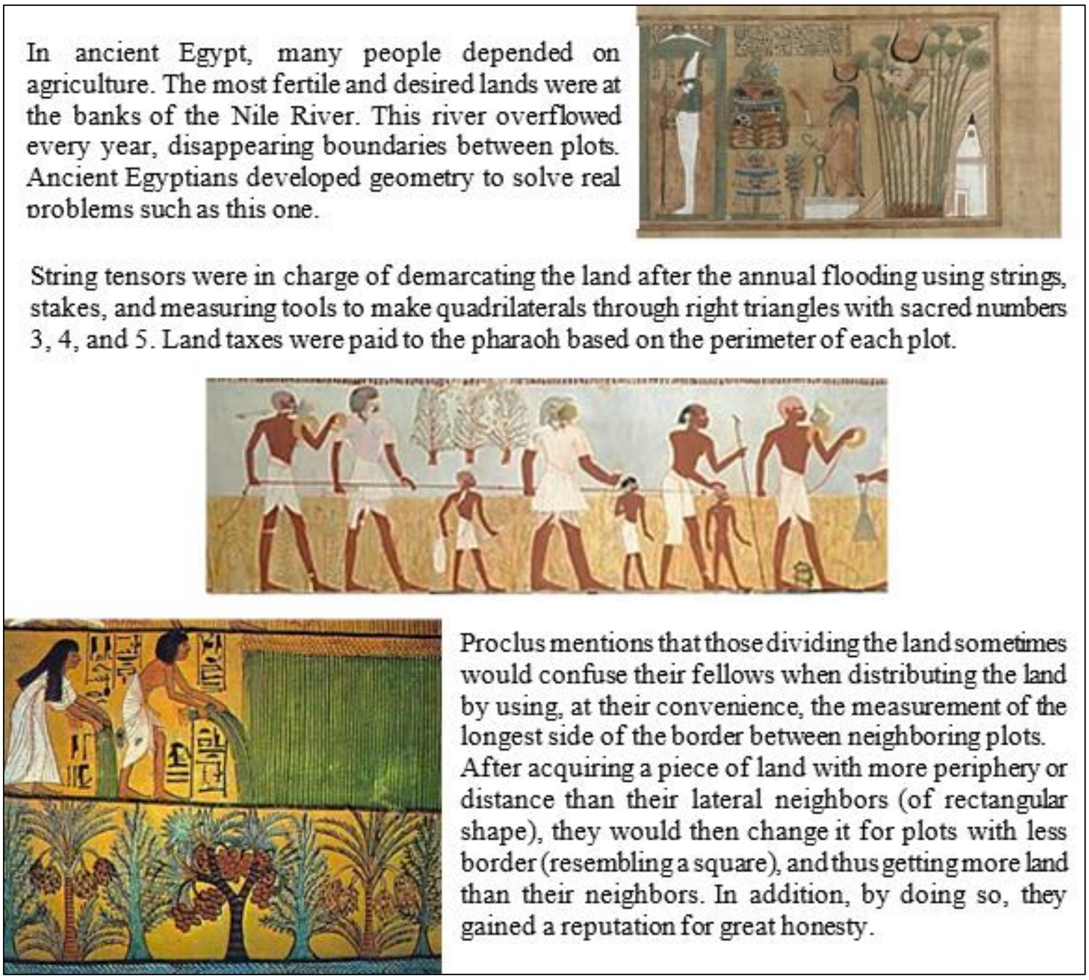

Figure 2. History of Proclus. Source: Salazar & Théry (2023, p. 5).

The task

The task called "Tensors", which is based on the story of Proclus, is presented to the teachers participating in the research with the support of images and texts, as shown in Figure 2, and they are asked to use the digital geoplane "Geoboard" (https://apps.mathlearningcenter.org/geoboard/) to make two quadrilaterals with right angles representing the land that the string tensors would delimit.

This task, unlike emblematic tasks (typical tasks presented in textbooks), takes aspects of the history of mathematics and has a different structure; it allows mobilizing the notion of the relation between area and perimeter of rectangles and squares by determining, for example, the reasons why some landowners changed the measurements of the sides of their land while keeping the same perimeter, but increasing its area.

Teachers are asked to use Geoboard to show why landowners modified the length and width of their lands while keeping the same perimeter recorded by the string tensors. Figure 3 shows a possible solution that teachers make using Geoboard.

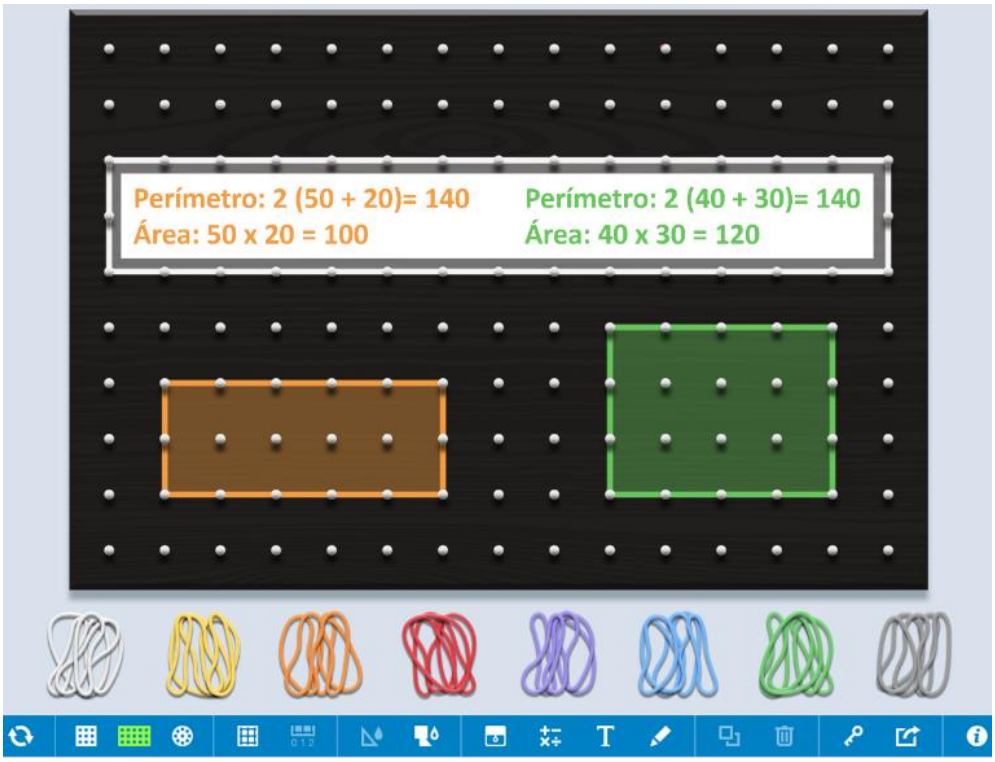

Figure 3. Geoboard representation. Source: Salazar & Théry (2023, p. 6).

The orange rectangle in the figure represents the original land of one of the landowners before the rise of the Nile in the story of Proclus. The green quadrilateral represents the quadrilateral that a landowner gets after the rise of the Nile by changing the measurements of the sides of his land and asking string tensors for a piece of land with the same perimeter as their original one.

In terms of MWS, this task promotes teachers' activation of the semiotic genesis by representing the plots as squares or rectangles. In addition, the task encourages the use of Geoboard as a digital artifact, the activation of the instrumental genesis, and, consequently, the activation of the vertical plane [Sem-Ins]. On the other hand, it also allows the activation of the discursive genesis based on the reference (concept of areas and perimeters) that teachers have in order to identify the area and perimeter of quadrilaterals and explain the relation between area and perimeter using mathematical properties. Thus, when using these properties to justify their representation, the [Sem-Dis] plane would be activated while the [Ins-Dis] plane is activated when using Geoboard to calculate areas and perimeters in order to look for a relation between them, "for the perimeter to be the same while increasing the area". A summary of the aforementioned is shown in Table 1.

Table 1. Task in Geoboard. Source: Adapted from Théry (2023, p. 98).

Task	Description	Geneses and Planes
Tensors	Representation of quadrilaterals described in the task as a figural representation.	Semiotic Genesis
	Construction of quadrilaterals with Geoboard according to the given measurements of the sides.	Instrumental Genesis
	Identification of the area and perimeter of quadrilaterals. Explanation of the relation between area and perimeter using mathematical properties.	Discursive Genesis
	Construction of quadrilaterals with Geoboard according to the given measurements.	[Sem-Ins] Plane
	Use of the properties of the described quadrilaterals to mathematically explain their representation.	[Sem-Dis] Plane
	Use of Geoboard to calculate areas and perimeters looking for a relation between them.	[Ins-Dis] Plane

The teachers solved the task (performing the role of students), and then reflected on pedagogical and technological aspects, which allowed them to demonstrate their knowledge and level of technology integration.

RESULTS AND DISCUSSION

The Tensor task was solved by the teachers in a similar way to what was previously planned (as shown in figure 3). In addition, the teachers stated that they would use this type of task and digital artefact to teach quadrilaterals because it is "more attractive, playful and integrative". For example, in the pedagogical discussion that took place after finishing the task, the teacher Sara commented that: "This story and technological tool can be used as an introduction with the students and they can learn from both". On the other hand, regarding the level of adaptation of technology for teaching mathematics, she is at the level of adaptation of technology for teaching mathematics. While the teacher Andrés observed: "I didn't know about this tool in digital form. I thought it was more for children... in concrete", this shows that, in relation to the levels of integration of technology in teaching, Andrés is at the level of acceptance of technology.

In the pedagogical discussion a question also emerged: *How does knowledge about the use of technology and the history of mathematics affect the teaching of quadrilaterals in the way I teach quadrilaterals?* teachers Sara and Andrés answered as follows:

Teacher Sara comment that "the use of technology and my understanding of the history of mathematics influences the way I do my lesson planning, i.e. the choice of problems and also the choice of the technological resource. So, I think, the use of some technological resource could be good or not ... this will depend on whether both my students and I know how to use it. That's why it is necessary to incorporate both elements progressively into the lessons to allow students to develop skills in the process.

While teacher Andrés expressed that "the use of technology could help students to solve tasks or problems of historical context about quadrilaterals or any other content as it is very attractive and at the same time simplifies the work... But this does not mean that other skills that we should develop in students such as the construction of geometric figures with the use of (non-digital) measuring instruments are left apart.

Furthermore, the designed task, which uses the mathematics history and the Geoboard as a digital artefact, also allows teachers to modify their class planning on quadrilaterals, for example, when we compare the didactic sequences in their initial planning (before the workshop) with the modified planning (after the workshop) it is evident that, for example, teacher Andrés modifies it by including Geoboard (see Appendix).

In this sense, the use of technology (Geoboard) and the history of mathematics, in the planning of his class on quadrilaterals, Andrés showed a greater interrelation between his technological knowledge and his MWS *idoine*, because he modified the planning by including this artefact in a task of exploration of the properties of quadrilaterals and also in another introductory task. In the case of teacher Sara, she presents few interactions between technological knowledge and her MWS *idoine*, since she only expresses her interest and possible use of this digital artefact for teaching, but she does not make meaningful changes when planning the teaching of quadrilaterals. Likewise, the use of technology in this task corresponds mainly to the Recognition and Acceptance levels of technology integration levels (Niess et al., 2009) because a task is transformed to the digital medium with material resources without a drastic change in methodology or use, but with advantages in the participants' attitude and some simplification in the use of the technological resource.

Also, in relation to the design of task "Tensors", Mendes (2023, p. 15) states that teachers create a historical dimension with respect to mathematics, and it is necessary to think about inserting it in the digital environment because, from that moment on, they become a human that is not human, but just a holographic human, i.e., a digital typographic human because, after the arrival of the press, we were reproduced typographically, but now we are reproduced in three dimensions as real models in a virtual digital environment because the virtual environment now offers dynamic movement, and we are a dynamic and sequential typographic human; therefore, we must think as if we were inside this virtual digital environment.

CONCLUSIONS

It is necessary, from the Didactics of Mathematics, to continue reflecting on the possible connections between digital technology and the use of the history of mathematics in mathematics teacher training because we consider that integrating the history of mathematics and technology in tasks on different mathematical contents has an influence on teachers' *idoine* MWS.

We consider that, although what was done in this work partially answered the initial questions, new discussions could be opened in the field of research groups in Latin America, since when in-service teachers use digital and classic tools separately, they have difficulties using them; however, when using them together in coordination, it allows them to make progress and achieve coherent results.

Likewise, we can admit the relevance of the reflections presented on the connection possibilities between digital technologies and the proposed uses of the history of mathematics in mathematics teaching, especially when we emphasize that this connection requires teachers and researchers to organize a theoretical-practical relation to integrate these two ways of approaching mathematics in schools.

Therefore, to use the history of mathematics to teach it in a digital environment requires two knowledge domains: a historical and a technological one. If we do not master the historical knowledge related to the development of concepts, properties and mathematical relations, we will not be able to move forward. The same will happen, if we only master the technical knowledge of digital technologies, but not their use in mathematics teaching.

It should also be noted that there is a number of published papers in the field of history of mathematics for teaching, but very few in the field of connections between digital technologies and the history of mathematics for didactic purposes that integrates historical knowledge and digital technological knowledge with a theoretical foundation supporting these works. For this reason, our challenge is precisely to build models based on our experiences and reflection to promote discussion. To this end, it is essential to create the required working groups and, from there, we can at least move forward towards broadening our reflections on the conceptual and didactic potential of the approach discussed in this article.

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REFERENCES

Artigue, M. (2011). Tecnología y enseñanza de las matemáticas: desarrollo y aportes de la aproximación instrumental. *Cuadernos de Investigación y Formación en Educación Matemática*. 6(8), p 13-33. https://revistas.ucr.ac.cr/index.php/cifem/article/view/6948/6634

Bowers, J. & Becky S. (2011). Using technology to explore mathematical relationships: a framework for orienting mathematics courses for prospective teachers. *Journal of Mathematics Teacher Education* 14:285–304 DOI 10.1007/s10857-011-9168-x.

Codina, A. & Romero, I. (2016). Entornos Tecnológicos y su Influencia en los Espacios de Trabajo Matemático. *Revista Bolema*, 30(54), 96-119. http://dx.doi.org/10.1590/1980-4415v30n54a05

Drijvers, P., Ball, L., Barzel, B., Heid, K., Cao, Y., Maschietto, M. (2016). *Uses of Technology in Lower Secondary Mathematics Education*. Springer Open. https://www.springer.com/gp/book/9783319336657

García-Cuéllar, D., Almouloud, S. & Salazar, J. V.F. (2019). Los cuadriláteros como saber a enseñar en el currículo peruano. *Revista Paranaese de Educacao Matemática*. https://doi.org/10.33871/22385800.2019.8.17.449-474

Gómez-Chacón, A. Kuzniak, L. Vivier (2016). El rol del profesor desde la perspectiva de los Espacios de Trabajo Matemático. *Boletim de Educação Matemática*, 30, 1-22, DOI: http://dx.doi.org/10.1590/1980-4415v30n54a01

Hernández-Sampieri, R., Fernández, C., & Baptista, P. (2014). *Metodología de la Investigación*. Editorial Mc Graw Hill Education. https://www.uca.ac.cr/wp-content/uploads/2017/10/Investigacion.pdf

Koehler, M., Mishra, P. & Cain, W. (2015). ¿Qué son los Saberes Tecnológicos y Pedagógicos? *Virtualidad, Educación y Ciencia*, (10), 9-23. https://www.punyamishra.com/wp-content/uploads/2016/08/11552-30402-1-SM.pdf

Kuzniak, A., Nechache, A. & Salazar, J. (16-18 de septiembre de 2020). Student Teachers' Geometric Work and Flexible Use of Digital Tools. *Mathematics Education in the Digital Age* (MEDA), Linz, Austria. https://hal.science/hal-03925304v1/file/MEDA3proceedings_2022.pdf

Kuzniak, A., Montoya-Delgadillo, E., & Richard, P. R. (2022). Mathematical Work in Educational Context. En *Mathematics education in the digital era*. https://doi.org/10.1007/978-3-030-90850-8

Kuzniak, A., Montoya, E. & Vivier, L. (2016). El espacio de trabajo matemático y sus génesis. Cuadernos de Investigación y Formación en Educación Matemática. En *Centro De Investigación y Formación en Educación Matemática* (Ed.), *11*(15), 235-249. http://www.centroedumatematica.com/Cuadernos/CuadernosCompletos/Cuaderno15.pdf

Kuzniak, A. & Nechache, A. (2021). On forms of geometric work: a study with pre-service teachers based on the theory of Mathematical Working Spaces. *Educational Studies in Mathematics*, 106, 271–289. https://doi.org/10.1007/s10649-020-10011-2

Kuzniak, A. & Richard, P. (2014). Espacios de trabajo matemático. Puntos de vista y perspectivas. Revista Latinoamericana de Investigación en Matemática Educativa. http://dx.doi.org/10.12802/relime.13.1741a

Kuzniak, A., Tanguay, D., & Elia, I. (2016). Mathematical Working Spaces in schooling: an introduction. *ZDM Mathematics Education*, 48(6), 721–737. http://dx.doi.org/10.1007/s11858-016-0812-x

Mendes, I. A. (2023). Sobre escritas ideográficas dinâmicas da história da Matemática. Revista de Matemática, Ensino e Cultura – *REMATEC,* (44), e2023010. https://doi.org/10.37084/REMATEC.1980-3141.2023.n44.pe2023011.id515

Mendes, I. A. (2021). Sobre processos criativos nas histórias da criação matemática. In: Pereira, A. C. C.; Martins, E. B. (Orgs.). *Investigações científicas envolvendo história da Matemática sob o olhar da pluralidade*. Curitiba: Editora CRV, p. 63-74.

Mendes, I. A. (2020). History for the teaching of mathematics transformation and mobilization of mathematical knowledge for school. Pedagogical Research, *5*(3) em0072, 01-10. https://www.pedagogicalresearch.com

Niess, M., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, 9(1), 4-24.

Salazar, J.V.F.; García-Cuéllar, D.; Vivas, J. L. & Peñaloza, T. N. (2023). Quadrilaterals in the Dynamic Figural Register Through Digital Technology. En *Perspectives and Trends in Education and Technology.* (pp. 419-430) SINGAPUR: Springer.: https://link.springer.com/chapter/10.1007/978-981-99-5414-8_38

Salazar, J.V.F.; Gaona, J. & Richard, P. (2022). Mathematical Work in the Digital Age. Variety of Tools and the Role of Geneses. En *Mathematical Work in Educational Context.* (pp. 165-209) CHAM: SPRINGER. Recuperado de: https://link.springer.com/chapter/10.1007/978-3-030-90850-8_8

Salazar, J.V.F. & Théry, M. C. (2023). Tecnología digital y el uso de la historia de la matemática en la enseñanza: construyendo conexiones. *Revista de Matemática, Ensino e Cultura* - REMATEC, 1(44), 1-8. https://www.rematec.net.br/index.php/rematec/article/view/511

Théry, M. (2023). Interacción de conocimientos tecnológicos con el espacio de trabajo matemático idóneo de profesores sobre cuadriláteros. Pontificia Universidad Católica del Perú. (Tesis), Lima, 150p. http://hdl.handle.net/20.500.12404/26050

APPENDIX

Example of planning class (original and modified didactic sequence) - teacher Andrés

Initial sequence:

Sequence of the strategy		Resource	Time
Start	**Aim and organization:** Apply the properties of quadrilaterals through the exercises proposed in your material. **Knowledge and skills:** What is a quadrilateral? what properties are fulfilled in a quadrilateral? how can we solve the cases presented? what strategies would you use to solve the cases presented? what are the rules that must be fulfilled in these figures? **Cognitive conflict:** How would you solve the following situation shown? The material to be used this week is presented.		5´
Process	**Organization and accompaniment: development of the class. Monitoring sheet** The session is conducted on the Meet platform over a period of 45 min. The properties of quadrilaterals are identified in the situations presented. Explanations will be given using the virtual blackboard. Previous knowledge will be identified, the fundamental concepts of quadrilaterals will be explained with examples and interactive work will be done with the students based on their solution strategies.	Memo-book	25´
Close	**Assessment:** At the end of the class, there is virtual feedback on the content being worked on. Observations and comments are made on students' resolutions. A rubric is used as a checklist.		10´

Modified sequence:

Sequence of the strategy		Resource	Time
Start	**Aim and organization:** Apply the properties of quadrilaterals through the exercises proposed in your material. **Knowledge and skills:**		5´

	What is a quadrilateral? what properties are fulfilled in a quadrilateral? how can we solve the cases presented? what strategies would you use to solve the cases presented? what are the rules that must be fulfilled in these figures? **Cognitive conflict:** The Geoboard is used to construct figures that satisfy the conditions for being quadrilaterals. How would you solve the following situation? You are presented with material on quadrilaterals. You are presented with a problem about quadrilaterals using elements from the history of mathematics.		
Process	**Monitoring and support: class development. Support sheet** The session takes place on the Meet platform over a period of 45 min. The fundamental concepts of quadrilaterals will be explained so that students can identify their properties using the virtual whiteboard and the Geoboard in context problems (use of history). Interactive work will be done with the students and their solution strategies will be discussed.	Memo-book	25´
Close	**Assessment:** At the end of the class, there is virtual feedback on the content being worked on. Observations and comments are made on students' resolutions. A rubric is used as a checklist.		10´

Índice Remissivo

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