Trends in Agricultural and Environmental Sciences
Pantanal Editora
10.46420/TAES. e240009

Automatic and Semi-automatic Analytic Hierarchy Process (AHP)

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Abstract: The Analytic Hierarchy Process (AHP) is the most commonly used method for solving multi-criteria decision-making problems worldwide. Although AHP offers many advantages for problems with several alternatives and criteria, the pairwise comparisons require considerable effort. State-of-the-art methodologies have demonstrated that AHP is suitable for automating decisions on tabular data. Considering it, this paper proposes a new approach to decision-making in both automated and semi-automated ways. For the automated approach, formulas are proposed for the normalization and computation of criteria weights. For the semi-automated approach, functions are proposed to convert normalized tabular data into values on the Saaty scale. These values are then used to automatically construct pairwise comparison matrices. This approach allows decision-makers to generate such matrices when necessary, thereby minimizing or even eliminating the effort required for pairwise comparisons. Simulations demonstrate the differences obtained depending on the use of the conversion function. Comparisons with state-of-the-art methods reveal that the proposed approach is compatible with existing methodologies. The viability of the proposed methodology is also explored through problems of selecting genotypes/varieties of agricultural crops, showing its viability in real problems. The obtained results show that the proposed approach produces results similar to other decision-making methods.

Keywords: Decision-making; Conversion Function; Pairwise matrix.

Received: 2024-06-05
Accepted: 2024-07-24
Published: 2024-07-25

Main Editors
Alan Mario Zuffo

For citation: Oliveira, B. R.; Duarte, M. A. (2024). Automatic and Semi-automatic Analytic Hierarchy Process (AHP). Trends in Agricultural and Environmental Sciences, (e240009), DOI: 10.46420/TAES.e240009

1. Introduction

Decision-making is a daily task for both people and machines programmed to perform specific actions. Some decisions are very complex, and wrong choices can lead to significant losses. Examples include decision-making related to logistics, supplier selection, and naval warships (Bulut and Duru, 2018; Kumar, Padhi, and Sarkar, 2019; Santos, Araujo Costa, and Gomes, 2021). The decision process involves analyzing available alternatives in relation to specific criteria and sub-criteria. In complex problems, multiple criteria must be considered, requiring the implementation of a multi-criteria decision-making (MCDM) approach (Khan and Ali, 2020; Shao et al., 2020; Nabeeh et al., 2019).

The Analytic Hierarchy Process (AHP) is the most widely used method to solve MCDM problems globally (Khan and Ali, 2020; Melo et al., 2021; Ho and Ma, 2018; Rigo et al., 2020; Sbai, Benabbou, and Berrado, 2020). Developed by Thomas L. Saaty in the early 1970s, AHP is suitable for solving complex decision-making problems (Terzi, 2019) and has been applied in numerous fields: engineering, civil works, economy, construction, agriculture, finance, forestry, earthquake hazards, food, geography, water, purchasing, IoT (internet of things), human error assessment, banking, history, maritime industry, navigation, profit and loss, green initiatives, manufacturing, strategy, quality, e-invoicing, supply chain, healthcare, environment, natural disasters, safety, sustainability, pipeline systems, automobiles, R&D, hydrogen energy technology, policy making, software, mathematics, organic management, tourism, transportation, landslides, waste management, entropy (Khan and Ali, 2020), and judiciary (de Oliveira, Oliveira, and Duarte, 2016). This extensive list demonstrates the reliability of AHP in decision-making. Additionally, between 1982 and 2018, there were 10,388 publications on AHP from the 20 most prolific regions (Yu et al., 2021).

Generally, AHP models an MCDM problem in a hierarchical structure (Franek and Kresta, 2014). Alternatives form the lowest hierarchy level and are pairwise compared concerning some criterion/sub-criterion at the highest level (Saaty and Vargas, 2012). Sub-criteria are also compared regarding the criteria (Krejčí and Stoklasa, 2018). At the top hierarchy level is the decision-making goal. These comparisons usually use the Saaty's Fundamental (SF) scale, which ranges from 1 to 9 (Saaty and Vargas, 2012). This scale allows the conversion of verbal judgments into numerical values, transforming the subjectivity of the decision-maker into objective values. In other words, AHP converts intangible considerations into tangible ones, turning a multidimensional scale problem into a one-dimensional scale problem (Aguarón et al., 2019).

Comparisons of alternatives and criteria result in a Pairwise Comparison (PC) matrix. Each PC matrix is associated with a local priority vector (LPV), obtained using eigendecomposition (eigenvalues and eigenvectors) (Saaty, 2003). These vectors represent the relative importance of objects at a lower level compared to those at a higher level, enabling the ranking of alternatives/criteria. Aggregating the LPVs provides the global priority vector (GPV), which indicates the importance of each alternative in relation to the goal (Lin and Kou, 2021; Aguarón et al., 2019). It is also possible to assess the consistency of the judgments, allowing for improvement if any PC matrix is inconsistent (Saaty and Vargas, 2012). This feature distinguishes AHP from other decision-making methods by providing coherence in its approach (Aguarón et al., 2019).

One advantage of the hierarchical structure in AHP is that it allows the decision-maker to focus on specific criteria and sub-criteria while making judgments (Franek and Kresta, 2014). Another advantage is its ease of handling both qualitative and quantitative data (Moeinaddini et al., 2010; Costa, Borges, and dos Santos Machado, 2016). Additionally, AHP's flexibility is demonstrated by its wide range of applications (Ho and Ma, 2018). Decision-making can be based on both knowledge and expertise about a subject and the analysis of numerical data, their relationships, and trends (Hutchinson, Alba, and Eisenstein, 2010). AHP combines these two types of information in a single approach. These factors make it the most applied methodology for solving MCDM problems.

However, despite its advantages, AHP requires considerable effort in problems with many alternatives and criteria due to the need for $k(k-1)/2$ comparisons (Saaty and Vargas, 2012), where k is the number of objects in a hierarchical level. Leal (2020) proposed a six-step approach that reduces the number of comparisons to $k-1$, assuming consistency in judgments. In environments with many comparisons, inconsistencies in judgments can occur because psychologists note that comparisons of many alternatives are often inaccurate (Franek and Kresta, 2014, citing Saaty, 1977). There is also the issue of missing comparison judgments, leading to an incomplete PC matrix (El Hefnawy and Mohammed, 2014). Supporting this, Saaty (1990) stated, "Comparisons of elements in pairs require them to be homogeneous or close with respect to the common attribute; otherwise, significant errors may be introduced into the measurement process. Additionally, the number of elements being compared must be small (not more than 9) to improve consistency and accuracy of measurement.".

Given these difficulties and considering that decision-makers in large databases may be biased (Hutchinson, Alba, and Eisenstein, 2010), this paper proposes a new approach to automatically obtain priority vectors without pairwise comparisons. This approach eliminates the effort of comparisons on the SF scale. The proposed methodology is suitable for decision-making based on tabular data and can be extended to big data applications for obtaining priority vectors automatically.

Additionally, a semi-automated approach is proposed where the PC matrix is generated from the data matrix of the problem. This intervention allows the decision-maker's subjectivity and expertise to be incorporated only in necessary judgments, reducing their workload while enabling intervention when needed. Functions are proposed to convert the normalized values of the data matrix to the SF scale, approximating the values of the ratios associated with judgments and maintaining the order obtained by the automated approach, only changing the magnitude of priorities.

This paper is organized as follows: Section 2 presents the fundamentals of AHP necessary for understanding the discussed concepts. Section 3 introduces the proposed approaches, discussing the automated and semi-automated methods using examples. Section 4 analyzes and discusses the proposed conversion functions and compares the proposed approaches with state-of-the-art methods, presenting an application in agricultural. Finally, Section 5 lists the conclusions and future work ideas.

2. Background

The AHP method consists of modeling a decision problem in a hierarchical structure. The goal is at the top, criteria[1] are in the intermediate level, and alternatives are in the lower level. Lower-level objects are pairwise compared in relation to higher-level objects. When comparing an object i with an object j, the decision maker must assign a score for this comparison. Saaty established that this score must be an integer between 1 and 9 (Saaty, 1990). These values are related to the degree of importance in the comparison. They are associated with verbal expressions as follows: 1 for equal; 3 for moderate; 5 for strong; 7 for very strong and 9 for extreme; 2, 4, 6 and 8 for intermediate values. However, if the comparison of object i with object j is assigned an integer k, then in the comparison of object j with object i, a score of $1/k$ is assigned, which is not an integer value. These values and degrees make up the SF scale (Saaty and Vargas, 2012).

The mentioned comparison score is written as $a_{ij}^{(m)} = \alpha$ where α or $1/\alpha$ belongs to SF scale, where (m) designates that the comparisons were performed in relation to the $m-th$ criterion. The result of the judgment (comparison) from objects i and j, represented by $a_{ij}^{(m)}$, is the ratio between the real value of object i and the one from object j, i.e., $a_{ij}^{(m)} = w_i^{(m)}/w_j^{(m)}$ in relation to the $m\, m-th$ criterion. However, the decision maker does not know the values $w_i^{(m)}$ and $w_j^{(m)}$. Therefore, only an $a_{ij}^{(m)}$ estimate is made. When assigning a value from the SF scale, if it occurs $w_i^{(m)}/w_k^{(m)} = (w_i^{(m)}/w_j^{(m)})(w_j^{(m)}/w_k^{(m)})$, for any i, j, k, i.e., $a_{ik}^{(m)} = a_{ij}^{(m)} a_{jk}^{(m)}$, then the judgment is said to be consistent. In this case, the object weight values are obtained by solving (Saaty, 2003) the Eigendecomposition problem (equation (1)) with a restriction (equation (2)):

$$\boldsymbol{A}^{(m)}\boldsymbol{w}^{(m)} = \lambda^{(m)}\boldsymbol{w}^{(m)} \text{ and} \tag{1}$$

$$\sum_{n=1}^{N} w_n^{(m)} = 1, \tag{2}$$

[1] AHP method also supports sub-criteria in the hierarchy. But for the purposes of this work sub-criteria will not be considered.

where $\boldsymbol{A}^{(m)} = a_{ij}^{(m)}$ is an $N \times N$ pairwise comparison (PC) matrix, $\boldsymbol{w}^{(m)}$ is the right eigenvector and $\lambda^{(m)}$ is the associated eigenvalue. Since $a_{ik}^{(m)} = a_{ij}^{(m)} a_{jk}^{(m)}$, from SF scale $a_{ii}^{(m)} = 1$ so $a_{ij}^{(m)} = 1/a_{ji}^{(m)}$. This is the reciprocal pairwise relation. For consistent judgments, the eigenvalue $\lambda^{(m)}$ is equal to N. In this case, the Eigendecomposition results in only a non-zero eigenvalue, since the matrix trace is equal to N. On the other hand, if there is inconsistency in the judgments, the values of the pairwise comparisons become $a_{ij}^{(m)} = e_{ij} w_i^{(m)} / w_j^{(m)}$, where e_{ij} is a perturbation. In this situation, the largest eigenvalue and eigenvector associated to it is taken as a solution of equation (1) (Saaty, 2008). To measure inconsistent judgments, the consistency index is used: $CI = (\lambda_{max} - N)/(N-1)$. Small inconsistencies are tolerated as long as $CI/RI < 0.1$, where RI is a previously calculated random index for various PC matrix dimensions (Saaty, 2008). In a decision problem with N alternatives and M criteria, there will be M $\boldsymbol{A}^{(m)}$ PC matrices and M local priority vectors (LPV) given by $\boldsymbol{v}_m = \left[w_1^{(m)}\ w_2^{(m)}\ \dots\ w_N^{(m)}\right]^{\boldsymbol{T}}$ relate to PC matrices $\boldsymbol{A}^{(m)}$, where $m = 1,2,\dots,M$. These vectors compose a matrix

$$\boldsymbol{V} = [\boldsymbol{v}_1\ \boldsymbol{v}_2 \dots \boldsymbol{v}_M] = \begin{bmatrix} w_1^{(1)} & w_1^{(2)} & \dots & w_1^{(M)} \\ w_2^{(1)} & w_2^{(2)} & \dots & w_1^{(M)} \\ \vdots & \vdots & \dots & \vdots \\ w_N^{(1)} & w_1^{(2)} & \dots & w_1^{(M)} \end{bmatrix}_{N\times M}. \quad (3)$$

The cross product of matrix $\boldsymbol{V}$ with the LPV of each criterion, i.e., $\boldsymbol{u} = [u_1\ u_2\ \dots\ u_M]$, gives the global priority vector (GPV) $\boldsymbol{x} = [x_1\ x_2\ \dots\ x_N]$, according to equation (4). Note that $\boldsymbol{u}$ is obtained according to equations (1) and (2), but for comparisons between the criteria in relation to the goal.

$$\boldsymbol{x} = \boldsymbol{uV}. \quad (4)$$

3. Proposed approach

The proposed approach is designed to apply AHP to tabular data. For this data structure, columns are the criteria and rows are the alternatives. Therefore, sub-criteria are not considered. This structure is the most common for organizing data, and it is present in: spreadsheets, relational databases, csv files, among others. As the criteria can be represented in different magnitude scales, data normalization is necessary. Two ways of normalization are proposed: 1) "the larger, the more preferable" (LMP); and 2) "the smaller, the more preferable" (SMP). Let $\boldsymbol{V} = (v_{ij}) \in \mathbb{R}^{N\times M}$ be a real data matrix with N alternatives and M criteria, where v_{ij} is the value of alternative i with respect to criterion j. In order to avoid negative values, a transformation in amplitude is proposed. It must be applied before normalization. Let $\boldsymbol{v}_m$ be the $m-th$ column of $\boldsymbol{V}$ data matrix. The amplitude transformation is given by $T_{\text{amp}}: \mathbb{R}^{N\times M} \to \mathbb{R}_+^{N\times M}$, equation (5), where $\mathbb{R}_+$ is the set of positive reals,

$$\boldsymbol{V}_m^* = T_{\text{amp}}(\boldsymbol{V}_m, \tau) = \boldsymbol{V}_m + |\min(\boldsymbol{v}_m)| + \tau, \quad (5)$$

where $\tau > 0$ is used to avoid the occurrence of null values. It is necessary so that when considering the ratios between these values, division by zero does not occur. After applying this transformation, a $\boldsymbol{V}^*$ matrix of positive real data is obtained. For LMP and SMP type criteria, the normalization is according to equations (6) and (7), respectively:

$$v'_{ij} = \frac{v^*_{ij}}{\sum_{k=1}^{K} v^*_{kj}}, \quad (6)$$

$$v'_{ij} = \frac{1}{v^*_{ij} \sum_{k=1}^{K} \frac{1}{v^*_{kj}}}, \quad (7)$$

resulting in a normalized data (ND) matrix $\boldsymbol{V'}$. Since the i-th row in $\boldsymbol{V'}$ sums to 1, then it already contains the weights of alternatives according to AHP theory. That is, ND matrix $\boldsymbol{V'}$ is the matrix from equation (3). It is now enough to determine the weights of each criterion, assigning the values of vector $\boldsymbol{u}$, equation (4), for the decision problem to be solved. Intuitively, we know that if the values of the alternatives vary slowly with respect to a certain criterion, then this criterion is less relevant. Therefore, it is proposed to the criterion weight to be given as a function of the mean deviation from the values of the alternatives in relation to that criterion, thus:

$$\boldsymbol{u}_m = \frac{1}{N} \sum_{n=1}^{N} |v'_{nm} - \overline{\boldsymbol{V'_m}}| \text{ with } m = 1,2,\dots,M \quad (8)$$

$$\boldsymbol{u}'_m = \frac{u_m}{\sum_{p=1}^{M} u_p}, \quad (9)$$

where $\overline{\boldsymbol{V'_m}}$ is the mean of the m-th column from ND matrix and $\boldsymbol{u}'_m$ is the normalized criterion weight.

3.1 Automated approach

The first proposed approach steps are: 1) apply the amplitude transformation when there are values less than or equal to zero; 2) normalize the data matrix considering the types of normalization given by equations (6) and (7); 3) obtain the criteria weights using equations (8) and (9); 4) calculate the priorities of the alternatives or the GPV solving the matrix product according to equation (4).

In order to illustrate the application of the proposed approach, the example of buying a house is considered. The data is broken down in Table 1, where the criteria are in the columns, and the values of alternatives are in the rows. This decision-marking problem was studied by Saaty & Vargas (2012).

Table 1. Data to decide on the purchase of a house.

	Price (US\$ × 10^3)	**Size (m²)**	**Renewal cost (US\$)**	**Style**
House 1	200	150	3000	Colonial
House 2	300	50	2000	Ranch
House 3	500	100	5500	Split Level

The first step in the proposed approach is to identify criteria such as LMP or SMP type, since in this scenario it is not necessary to apply the amplitude transformation. The criterion ‘Size’ is of the first type, while ‘Price’ and ‘Renewal Cost’ are of the second type. On the other hand, the ‘Style’ criterion is non-numeric (or intangible) (Saaty & Vargas, 2012), therefore, it is necessary to assign numerical values to the verbal descriptions. The following encoding is proposed: Colonial = 1; Ranch = 2 and Split Level = 3. This encoding implicitly gives more weight to the ‘Split Level’ style, then ‘Ranch’ and lastly ‘Colonial’. Such assignment will also affect the weights obtained from the normalized data. In this case, the decision maker must choose whether the criterion will be of the LMP or SMP type, since there is no magnitude associated to the criterion values that can be ordered. So, let the 'Style' criterion be of LMP type, due to the assigned values. In this way, after applying the normalization formulas, equations (6) and (7), we obtain the matrix

$$\boldsymbol{V}' = \begin{bmatrix} 0.4839 & 0.5000 & 0.3284 & 0.1667 \\ 0.3226 & 0.1667 & 0.4925 & 0.3333 \\ 0.1935 & 0.3333 & 0.1791 & 0.5000 \end{bmatrix}.$$

The first line in $\boldsymbol{V}'$ means that 'Price', 'Size', 'Renewal Cost' and 'Style' criteria have weights 0.4839, 0.5, 0.3284 and 0.1667, respectively, in relation to House 1. The values of the other rows in $\boldsymbol{V}'$ are similarly interpreted. The weights of the criteria 'Price', 'Size', 'Renewal Cost' and 'Style', calculated according to equations (8) and (9), are: $\boldsymbol{u}' = [0.2341 \;\; 0.2592 \;\; 0.2476 \;\; 0.2592]$. Such weights mean that none of the criteria is much more relevant than the others. Finally, the weights (priorities) of the alternatives are given by $\boldsymbol{x} = \boldsymbol{u}'\boldsymbol{V}'^T = [0.3673 \;\; 0.3270 \;\; 0.3056]$. Thus, the choice would be House 1.

Although the decision maker intervened in encoding the values of 'Style' criterion, the first approach is automated. In other words, the expertise and subjectivity of the decision maker were not considered in the choice of alternatives. But this is one of the advantages of AHP, i.e., it allows the decision maker's subjectivity to be aggregated in the decision-making process. Additionally, Saaty mentions that "In decision-making the priority scales are derived objectively after subjective judgments are made." (Saaty, 2008). He also claims that AHP "enables us to cope with the intuitive, the rational, and the irrational, all at the same time, when we make multicriteria and multiactor decision." (Saaty, 1986). On another occasion he says "the purpose of decision-making is to help people make decisions according to their own understanding". Besides that, Costa & Vansnick (2008) believe that "the elicitation of pairwise comparison judgements and the possibility of expressing them verbally are cornerstones of the popularity of AHP.". For these reasons, in the next section it is presented a new approach to automatically generate pairwise comparison (PC matrices) from ND matrix, in order to enable the decision maker intervention.

3.2 Semi-automated approach

In the second approach, it is proposed to modify the automated approach so that it is possible to incorporate the decision maker subjectivity when he/she thinks it is necessary. It will be done acting in reverse to the common AHP implementation, i.e.: from the ND matrix we will build the PC matrices. With the PC matrices in hand, the decision maker can modify those comparisons that deem necessary, and after solving the decision problem as usually done, i.e., by means of Eigendecomposition, using equation (1).

For this, we propose the construction of conversion functions, which convert normalized values to ratios on SF scale. According to Saaty & Vargas (2012), approximating the ratios coming from the judgments to the nearest integer is a central fact in AHP. The mentioned functions were first proposed by Oliveira et al. (2019) and Oliveira, Duarte and Vieira Filho (2022), but only in the context of selecting machine learning models. In those works, normalized data were not considered and the AHP additive models were implemented. According to the theory established by Saaty, for the m-th criterion, each element from $\boldsymbol{A}^{(m)}{}_{N\times N}$ PC matrix is given by $a_{ij}^{(m)} = w_i^{(m)}/w_j^{(m)}$. From the proposed approach, the m-th column from $\boldsymbol{V}'$ ND matrix is $\boldsymbol{v}'_m = \left[w'^{(m)}_1 \;\; w'^{(m)}_2 \;\; \dots \;\; w'^{(m)}_N\right]^T$. Since $w_i^{(m)}/w_j^{(m)}$ is the ratio between values ranging from 1 to 9, then $w'^{(m)}_i$ gets closer to $w_i^{(m)}$ given that values are normalized. Therefore, using ND matrix values, it is possible to obtain the PC matrices employing an appropriated conversion function.

Let $r_{ij}^{(m)} = {w'}_i^{(m)}/{w'}_j^{(m)}$ be the ratio between the i-th and the j-th elements from the ND matrix related to the m-th criterion (or m-th column from this matrix). To convert the $r_{ij}^{(m)}$ ratio to SF scale, it is proposed the application of functions (10) to (13) for each $r_{ij}^{(m)}$ ratio separately.

$$f\left(r_{ij}^{(m)}\right) = \begin{cases} 1 & \text{, if } r_{ij}^{(m)} \geq 1 \\ -1, & \text{if } r_{ij}^{(m)} < 1 \end{cases}, \tag{10}$$

$$s_{ij}^{(m)}\left(r_{ij}^{(m)}\right) = \left\lceil {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} \right\rceil, \tag{11}$$

$$l_{ij}^{(m)} = \begin{cases} 1, \text{if } s_{ij}^{(m)} < 1 \\ 9, \text{if } s_{ij}^{(m)} > 9 \\ s_{ij}^{(m)}, \text{otherwise} \end{cases}, \tag{12}$$

$$C_{ij}^{(m)}\left(l_{ij}^{(m)}, r_{ij}^{(m)}\right) = {l_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)}, \tag{13}$$

where $\left\lceil {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} \right\rceil$ is the ceiling function, which returns the smallest integer greater than ${r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)}$, i.e., $\left\lceil {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} \right\rceil = \min\left\{p \in \mathbb{Z}_+; p \geq {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)}\right\}$[2].

Thus, the $\boldsymbol{A}^{(m)}$ PC matrix is formed by elements $C_{ij}^{(m)}$ resulting from the conversion. Function (10) has the purpose of always returning 1 or -1. Function (11) returns an approximation of the ratio $r_{ij}^{(m)}$, eliminating the decimal part. Function (12) limits the output from function (11) to the interval [1, 9], so that the conversion can be performed according to SF scale. Finally, function (13) keeps the output from (12) or reverses it, depending on whether $r_{ij}^{(m)}$ is greater or less than 1. It is necessary because $a_{ij}^{(m)} = 1/a_{ji}^{(m)}$, according to AHP theory. The algorithm for implementing the ratio conversion to SF scale is shown in Algorithm 1[3].

Input:

$\boldsymbol{v'}_m$: m-*th* column of the ND matrix

Output:

$\boldsymbol{C}^{(m)}$: PC matrix

1. Function $f(r)$:
2. If $r \geq 1$

[2] Although ceiling function is defined for $p \in \mathbb{Z}$, due to amplitude transformation application, in this paper only $p \in \mathbb{Z}_+$ is considered.
[3] Source code is available on https://github.com/brunobro/ahptd

```
    return 1
Else
    return −1
end If
end Function

N ← number of elements in v'_m
C ← matrix with all elements equal to 1

For i = 1 to N do
    For j = i + 1 to N − 1 do
        r ← v'_m(i)/v'_m(j)
        s ← r^f(r)
        s ← ceil(s)  (or s ← floor(s) or s ← round(s))
        l ← s
        If s < 1
            l ← 1
        Else If s > 9
            l ← 9
        end If
        C(i, j) ← l^f(r)
        C(j, i) ← 1/l^f(r)
    end For
end For
```

Algorithm 1. Semi-automated approach algorithm to generate PC matrix.

As an example, consider three alternatives with weights ${v'}_1^{(m)} = 0.5$, ${v'}_2^{(m)} = 0.2$ and ${v'}_3^{(m)} = 0.1$, in relation to some criterion m, obtained from a ND matrix. The ratios are: $r_{12}^{(m)} = 0.5/0.2 = 2.5$, $r_{13}^{(m)} = 0.5/0.1 = 5$, $r_{21}^{(m)} = 0.2/0.5 = 0.4$, $r_{23}^{(m)} = 0.2/0.1 = 2$, $r_{31}^{(m)} = 0.1/0.5 = 0.2$ and $r_{32}^{(m)} = 0.1/0.2 = 0.5$. Applying functions (10) and (11) to these ratios results in: $s_{12}^{(m)} = 2$, $s_{13}^{(m)} = 5$, $s_{21}^{(m)} = 2$, $s_{23}^{(m)} = 2$, $s_{31}^{(m)} = 5$ and $s_{32}^{(m)} = 2$. Since there are no values smaller than 1 or greater than 9, function (12) will not change the obtained values. The converted values using function (13) are: $C_{12}^{(m)} = 2$, $C_{13}^{(m)} = 5$, $C_{21}^{(m)} = 1/2$, $C_{23}^{(m)} = 2$, $C_{31}^{(m)} = 1/5$, $C_{32}^{(m)} = 1/2$. These values result in the PC matrix

$$\boldsymbol{A}^{(m)} = \begin{bmatrix} 1 & 2 & 5 \\ 1/2 & 1 & 2 \\ 1/5 & 1/2 & 1 \end{bmatrix},$$

whose LPV is $\boldsymbol{w}^{(m)} = [0.5954 \quad 0.2764 \quad 0.1283]$ and the maximum eigenvalue is $\lambda_{max} = 3.0055$. Therefore, the consistency ratio is 1.8092×10^{-3}, then the PC matrix is consistent.

Although in function (11) the ceiling function was used, one could also use the floor function, i.e., $\left\lfloor {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} \right\rfloor = \max\left\{p \in \mathbb{Z}_+;\ p \leq {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)}\right\}$, or just return the nearest integer from ${r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)}$, defined as:

$$\left[\!\left[{r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} \right]\!\right] = \left\{p \in \mathbb{Z}_+; \left| {r_{ij}^{(m)}}^{f\left(r_{ij}^{(m)}\right)} - p \right| < 0.5\right\}.$$

The difference between the implementations will be explored in the next section, where the values of the ratios ranging from 0.01 to 9.9 are evaluated, which are converted using the mentioned functions.

In order to compare the automated and semi-automated approaches, equations (10) to (13) are applied to the ND matrix obtained from Table 1, resulting in the following PC matrices and LPV:

$$\boldsymbol{A}^{(1)} = \begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix}, \boldsymbol{w}^{(1)} = [0.5396 \ \ 0.2970 \ \ 0.1634],$$

$$\boldsymbol{A}^{(2)} = \begin{bmatrix} 1 & 3 & 2 \\ 1/3 & 1 & 1/2 \\ 1/2 & 2 & 1 \end{bmatrix}, \boldsymbol{w}^{(2)} = [0.5396 \ \ 0.1634 \ \ 0.2970],$$

$$\boldsymbol{A}^{(3)} = \begin{bmatrix} 1 & 1/2 & 2 \\ 2 & 1 & 3 \\ 1/2 & 1/3 & 1 \end{bmatrix}, \boldsymbol{w}^{(3)} = [0.2970 \ \ 0.5396 \ \ 0.1634],$$

$$\boldsymbol{A}^{(4)} = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/2 \\ 3 & 2 & 1 \end{bmatrix}, \boldsymbol{w}^{(4)} = [0.1634 \ \ 0.2970 \ \ 0.5396].$$

Arranging the LPV as columns of a new alternatives weight matrix, we obtain:

$$\boldsymbol{V}' = \begin{bmatrix} 0.5396 & 0.5396 & 0.2970 & 0.1634 \\ 0.2970 & 0.1634 & 0.5396 & 0.2970 \\ 0.1634 & 0.2970 & 0.1634 & 0.5396 \end{bmatrix}.$$

Considering the weights of the criteria previously obtained, the priorities of alternatives are given by

$$\tilde{\boldsymbol{x}} = \boldsymbol{u}\boldsymbol{V}'^T = [0.3820 \quad 0.3224 \quad 0.2955], \tag{14}$$

where $\tilde{\boldsymbol{x}}$ is the estimated GPV obtained using conversion functions (10) and (11). For this example, all consistent indices are equal to 8.8488×10^{-3}. Therefore, all PC matrices are consistent.

An analogous result is obtained by employing the nearest integer in conversion function (11). On the other hand, when using the floor function the obtained GPV is $\boldsymbol{x} = [0.3364 \quad 0.3231 \quad 0.3405]$ and the consistency ratios are approximately 0.051, 0.051, 0.017 and 0.017 for the PC matrices associated to criteria 'Price', 'Renewal', 'Size' and 'Style', respectively. Therefore, the PC matrices also are consistent, although the priorities values and their order are different when compared to results obtained using ceiling and nearest integer functions.

It is observed that the alternative weights obtained by applying the conversion function (semi-automated approach) are different from those obtained by automated approach. This result is expected, since the conversion to the Saaty' Scale approximates the ratios from the ND matrix. Even so, in the GPV, the orders of the alternatives are maintained, except for floor function application. The approximation error can be calculated as $\sum_{q=1}^{3}\left(x_q - x_q\right)^2 = 0.0756$, considering the results obtained with the use of the ceiling function.

Although the semi-automated approach approximates the values of the weights of the alternatives, it has the advantage of allowing the decision maker to change those PC matrices that deems pertinent. On the other hand, the automated approach does not allow this interaction. As an example, note that for the 'Style' criterion a coding scheme was chosen. But this criterion is difficult to codify properly as the characteristics involved are very broad. Therefore, the subjectivity of the decision maker can be used. Thus, the $\boldsymbol{A}^{(4)}$ matrix can be changed, for example, considering:

$$\boldsymbol{A}^{(4)} = \begin{bmatrix} 1 & 1/3 & 1/6 \\ 3 & 1 & 1/2 \\ 6 & 2 & 1 \end{bmatrix},$$

giving a lot more weight to the 'Split Level' style. Meanwhile, PC matrices obtained by the semi-automated approach are maintained. In this case the GPV would be $\boldsymbol{x} = [0.3656 \quad 0.3232 \quad 0.3112]$ using ceiling function, causing House 1 to be chosen, but, in this case, House 3 was more important when compared with the results in equation (14).

It is important to highlight that the criteria weights ($\boldsymbol{u}$ vector values) were obtained by applying equations (8) and (9) on the ND matrix and not on the matrix obtained after the pairwise comparisons. Alternatively, criteria weights could also be used in the proposed conversion functions, in the case where the decision maker wants to change the judgments. Let's look at this case. The original vector $\boldsymbol{u}$, obtained from the ND matrix, is equal to $[0.2341 \;\; 0.2592 \;\; 0.2476 \;\; 0.2592]$. Applying the conversion functions to this vector, equations (10) to (13), we have the following PC matrix and LPV:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1/2 & 1/2 & 1/2 \\ 2 & 1 & 2 & 1 \\ 2 & 1/2 & 1 & 1/2 \\ 2 & 1 & 2 & 1 \end{bmatrix}, \boldsymbol{u} = [0.1404 \quad 0.3300 \quad 0.1996 \quad 0.3300].$$

Thus, this result shows that the criteria weights, obtained using the conversion functions, are compatible with those obtained when considering only the ND matrix, as the selection order was not affected, but only the magnitude of the judgments and, consequently, the weights priorities.

4. Results and Discussion

4.1 Analysis of the conversion functions

As mentioned in the previous section, the conversion function can be implemented using the 'ceiling', 'floor' or 'nearest integer' approximation functions. Each of these functions generates different outputs, privileged by certain values. In order to understand their behavior, the graphs in Figures 1 and 2 illustrate the outputs (Converted ratio) when the ratio ranges from 0.01 to 9.9.

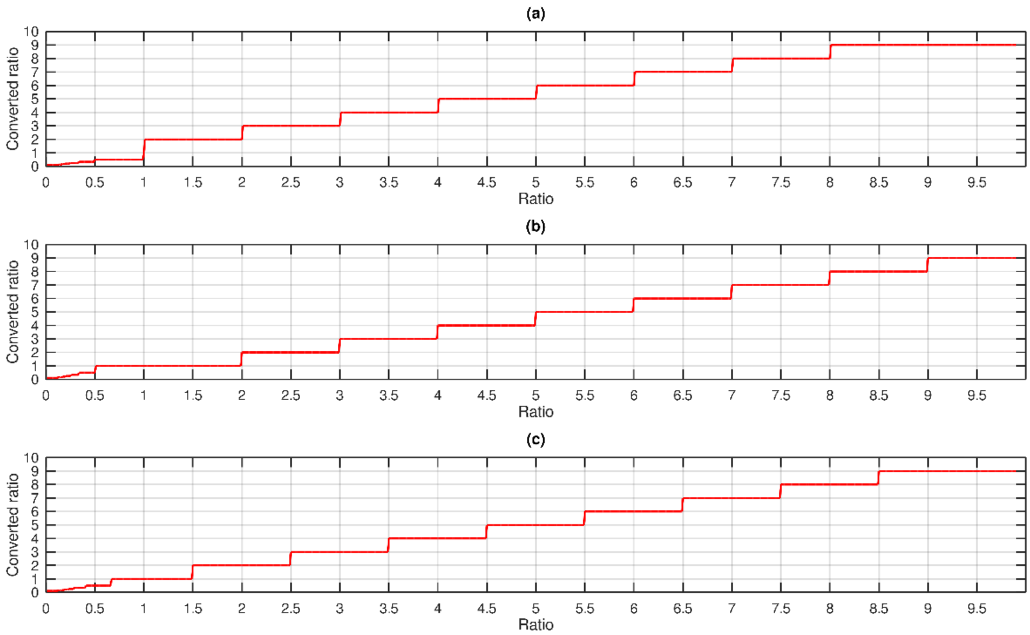

Figure 1. Conversion function output using: (a) ceiling, (b) floor and (c) nearest integer.

Figure 2 illustrates only converted ratio ranging from 0.01 to 1, just for the best visualization, since in Figure 1 it is not possible to perceive converted ratio values less than 1.

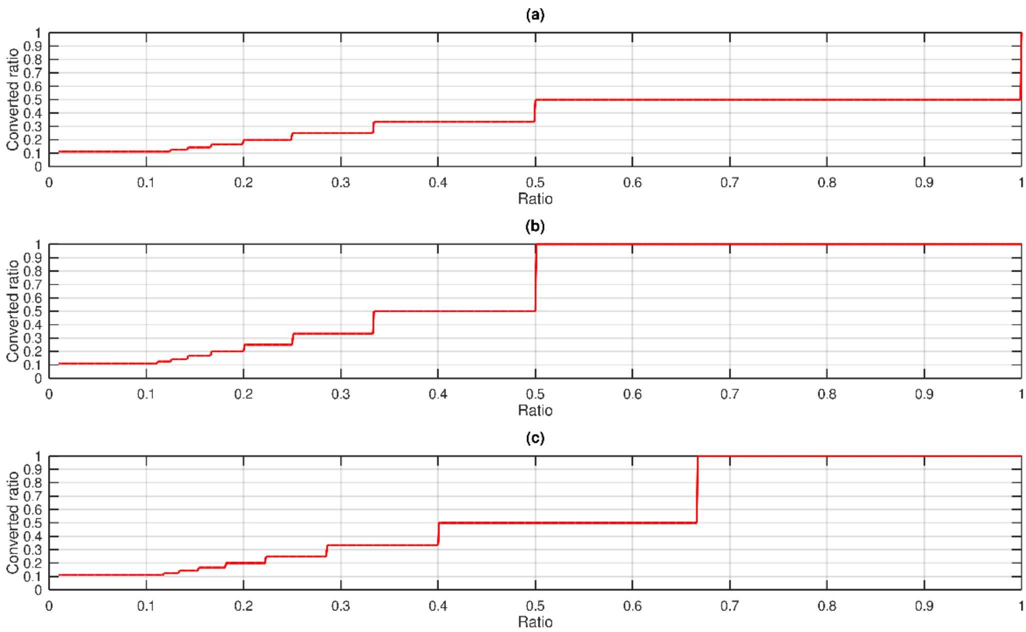

Figure 2. Conversion function output in the range from 0.01 to 1 using: (a) ceiling, (b) floor and (c) the nearest integer.

Note in the graphs in Figure 1 that the ceiling function, Figure 1 (a), converts more ratios to 9 than the other functions. On the other hand, the floor function, Figure 2 (b), converts more values to 1. The only output equal to 1 returned by the ceiling function occurs when the ratio is also equal to 1. For example, if two alternatives have approximated weights, such as $w_1 = 0.5$ and $w_2 = 0.4$, then the ceiling, floor and nearest integer returned outputs 2, 1 and 1, respectively, considering the ratio $w_1/w_2 = 0.5/0.4 = 1.25$. In other words, the floor and nearest integer functions favor the verbal judgment 'equal importance'. While the ceiling function will also return this same judgment, but as an intermediate value between 'moderate importance'. Consequently, for values greater than 8, the ceiling function will privilege the verbal judgment 'extremely important', while the floor function will only return this judgment if the value is greater than 9. On the other hand, the nearest integer function will return this judgment for values greater than 8.5.

Figure 2 also reveals that, regardless of the chosen function, if the ratio is less than 0.1 then it is converted to $1/9$, as equation (10) will consider the value $1/\alpha$ which will always be greater than 10, for $\alpha < 0.1$. Then the conversion function (11) will limit it to 9 while the function (12) returns $1/9$.

Therefore, the decision has maker to choose which judgments will be privileged, selecting the appropriate conversion function among those mentioned. Bearing in mind that depending on the choice, orders and values of alternatives may be different, as exemplified in the example in the previous section.

4.2 Analysis of the consistency

As mentioned earlier, pairwise comparison matrices can be inconsistent if $CI/RI < 0.1$. However, the proposed conversion function (11) does not depend on human judgments, which are the source of inconsistency. In order to verify if the application of this function generates inconsistent matrices, an experiment is proposed, as following: (1) generate a thousand random matrices varying their dimensions from 3 to 15; (2) apply the conversion function (11) for each matrix; (3) calculate the consistency index.

Figure 3 shows a box plot from one thousand consistency indices for pairwise comparison matrices with different dimensions.

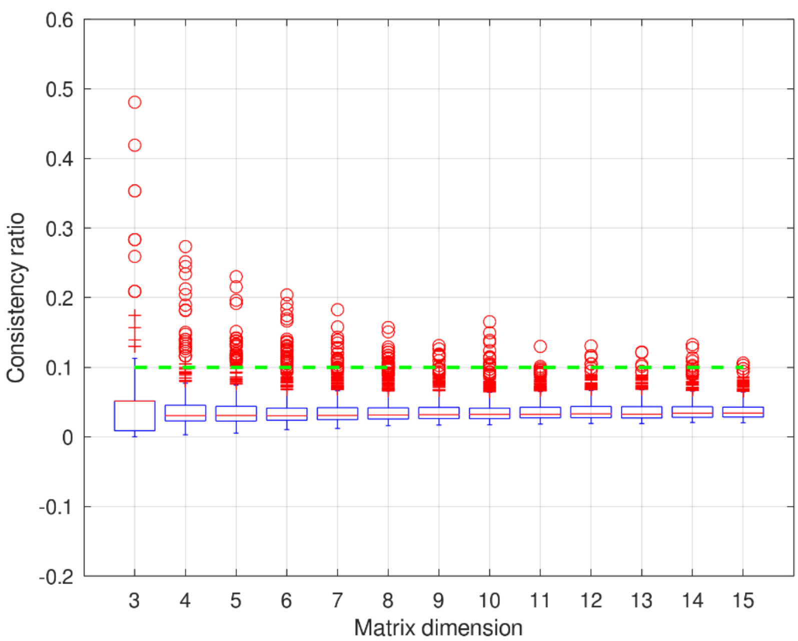

Figure 3. Consistency ratio for different matrix dimensions. The dotted line represents the threshold for deciding on consistency.

The results in Figure 3 show that among the thousands of generated matrices, some of them are inconsistent. Furthermore, the higher the matrix order, the closer to the 0.1 threshold the consistency indices are, although more inconsistent matrices were detected. It indicates that the proposed conversion function is suitable for big data applications, where many elements in the pairwise comparison matrices are expected. Additionally, as the proposed approach can also have interference from the decision maker, cases of inconsistency can be corrected. However, for the automated approach, this behavior can be a disadvantage.

4.3 Comparison with a state-of-the-art approach

In order to validate the proposed approach, in this section it will be compared with that presented by Santos, Araujo Costa and Gomes (2021), which is also suitable for application in tabular data. However, that approach does not generate PC matrices as the semi-automated proposed approach.

Table 2 shows the data used to choose among the available models of warships in the Brazilian Navy. Unlike the previous example, in this case the criteria are arranged in rows, just to better adapt to the text layout.

Table 2. Decision matrix for choosing warship model. Source: Santos, Araujo Costa and Gomes (2021).

	Model 1	**Model 2**	**Model 3**
Action Radius (C1)	4000	9330	10660
Fuel Endurance (C2)	11	26	30
Autonomy (C3)	30	25	35
Primary Cannon (C4)	25	25	120
Secondary Cannon (C5)	1	2	2
AAW Missiles (C6)	0	1	1
Initial Cost (C7)	290,000,000	310,000,000	310,000,000
Life Cycle Cost (C8)	592,000,000	633,000,000	633,000,000
Construction Time (C9)	6	8	8

In order to compare the criteria weights, Table 3 shows the values obtained by Santos, Araujo Costa and Gomes (2021) and our automated and semi-automated approaches.

Table 3. Comparison between criteria weights.

Criterion	**Criterion weight**		
	Santos, Araujo Costa and Gomes (2021)	**Automated approach**	**Semi-automated approach for $\tau = 0$**
Action Radius (C1)	0.126	0.126	0.105
Fuel Endurance (C2)	0.129	0.127	0.105
Autonomy (C3)	0.048	0.042	0.086
Primary Cannon (C4)	0.278	0.281	0.206
Secondary Cannon (C5)	0.099	0.100	0.072
AAW Missiles (C6)	0.248	0.251	0.152
Initial Cost (C7)	0.011	0.011	0.090
Life Cycle Cost (C8)	0.011	0.011	0.090
Construction Time (C9)	0.050	0.050	0.090

Using only the ND matrix and the criteria weights shown in Table 3 for automated approach, results in the following priorities for Models 1, 2 and 3, respectively: 0.1452, 0.3394 and 0.5155. The approach presented by Santos, Araujo Costa and Gomes (2021) got the priorities: 0.1465, 0.3390 and 0.5144, respectively. Therefore, these three approaches perform very similar results.

On the other hand, the proposed semi-automated approach consists in generating the PC matrices from the ND matrix. For this, the amplitude transformation, equation (5), can be applied for C6 criterion, since there exists a null value. As this transformation depends on the τ parameter, variations of it will be considered. Table 4 shows the priorities obtained for fixed τ values.

Table 4. Priorities obtained for different **τ** values.

τ parameter	Model 1	Model 2	Model 3
0.1	0.1663	0.3022	0.5315
0.2	0.1738	0.2967	0.5295
0.3	0.1765	0.2954	0.5281
0.4	0.1809	0.2927	0.5264
0.5	0.1876	0.2888	0.5235

It is noted in Table 4 that all τ values generate priorities compatible with those obtained by the aforementioned approaches. Namely, the ordering of alternatives is the same, although the weights are different. This result is also due to the fact that, when applying the amplitude transformation, the criteria weights are also changed. Figure 4 shows the weights for some values of τ.

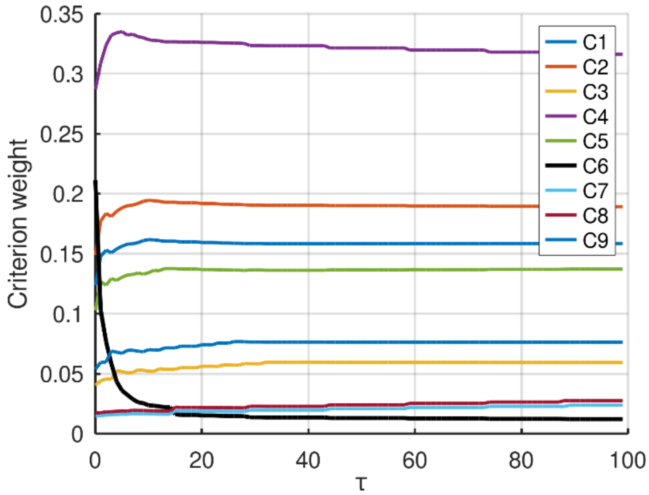

Figure 4. Variation of criteria weights for different **τ** values.

Note in Figure 4 that the higher the value of τ, the less weight is given to C6 criterion. Consequently, more weights are distributed to some criteria. This is because the mean deviation of the normalized values also changes. For example, given a matrix

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 1 \\ 0.1 & 1.1 & 1.1 \\ 0.9 & 1.9 & 1.9 \end{bmatrix},$$

where de second and third rows are obtained when adding the values 0.1 and 0.9 to the first row, respectively. Normalizing individually each row by its sum, results in the matrix

$$\mathbf{D'} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.0434 & 0.4782 & 0.4782 \\ 0.1915 & 0.4043 & 0.4043 \end{bmatrix}.$$

In this case, the weights related to the first, second and third rows of matrix $\mathbf{D}'$, using equations (8) and (9), are 0.4357, 0.3789 and 0.1854, respectively. This result shows that the greater is the value added to the criterion, the lower is its weight. Another observed consequence is that the larger the τ parameter, the more weight is given to the first alternative, as shown by the weights in the first column of matrix $\mathbf{D}'$. This result is very clear in Table 4 in relation to Model 1.

Finally, the last analysis shows that τ parameter must be chosen carefully. Keeping in mind that it is only necessary if there are values less than or equal to zero in the data matrix.

Other comparison is performed with the approach presented by Alelaiwi (2019) in the scenario of evaluating the database platform. The normalized data is shown in Table 5. The alternatives are: 'DaDaBIK', 'DataFlex', 'Oracle application express' and 'FileMaker'. The criteria are: 'Usability', 'Portability' and 'Supportability'. These values are obtained employing the classical AHP approach. Unlike the comparison performed earlier, in this case there are more alternatives than criteria.

Table 5. Normalized data matrix presented by Alelaiwi (2019).

Database platform	Usability	Portability	Supportability
DaDaBIK	0.059	0.0053	0.0255
DataFlex	0.020	0.0427	0.0255
Oracle application express	0.150	0.0061	0.1900
FileMaker	0.165	0.0455	0.2680

Table 6 shows comparisons among the results obtained by Alelaiwi (2019) and the semi-automated approach application using ceiling function.

Table 6. Comparisons: Alelaiwi (2019) and semi-automated.

Criterion	Criterion weight	
	Alelaiwi (2019)	**Semi-automated approach**
DaDaBIK	0.0868	0.0723
DataFlex	0.0882	0.1590
Oracle application express	0.3460	0.2477
FileMaker	0.4780	0.5208

According to the priorities shown in Table 6, both approaches returned the same database platform selection order. However, the proposed semi-automated approach gave more weight (proportionally) to the 'DataFlex' alternative, even without intervention in the generated PC matrices. Furthermore, all consistency ratios imply that the generated matrices are consistent.

4.4 Applications in agricultural

This section explores two real-world applications of the proposed approach in agricultural selection. Both experiments focus on identifying genotypes resistant to abiotic stress, specifically water limitations.

The first experiment evaluated 70 soybean genotypes under two water stress conditions: saline and drought. The Manhattan distance metric was used to quantify the impact of stress on various variables compared to a control environment. These distances were then fed into the TOPSIS decision-making method to identify cultivars with the smallest distances, indicating minimal stress impact (De Oliveira et al., 2022). Essentially, shorter distances signified greater stress tolerance in the genotype.

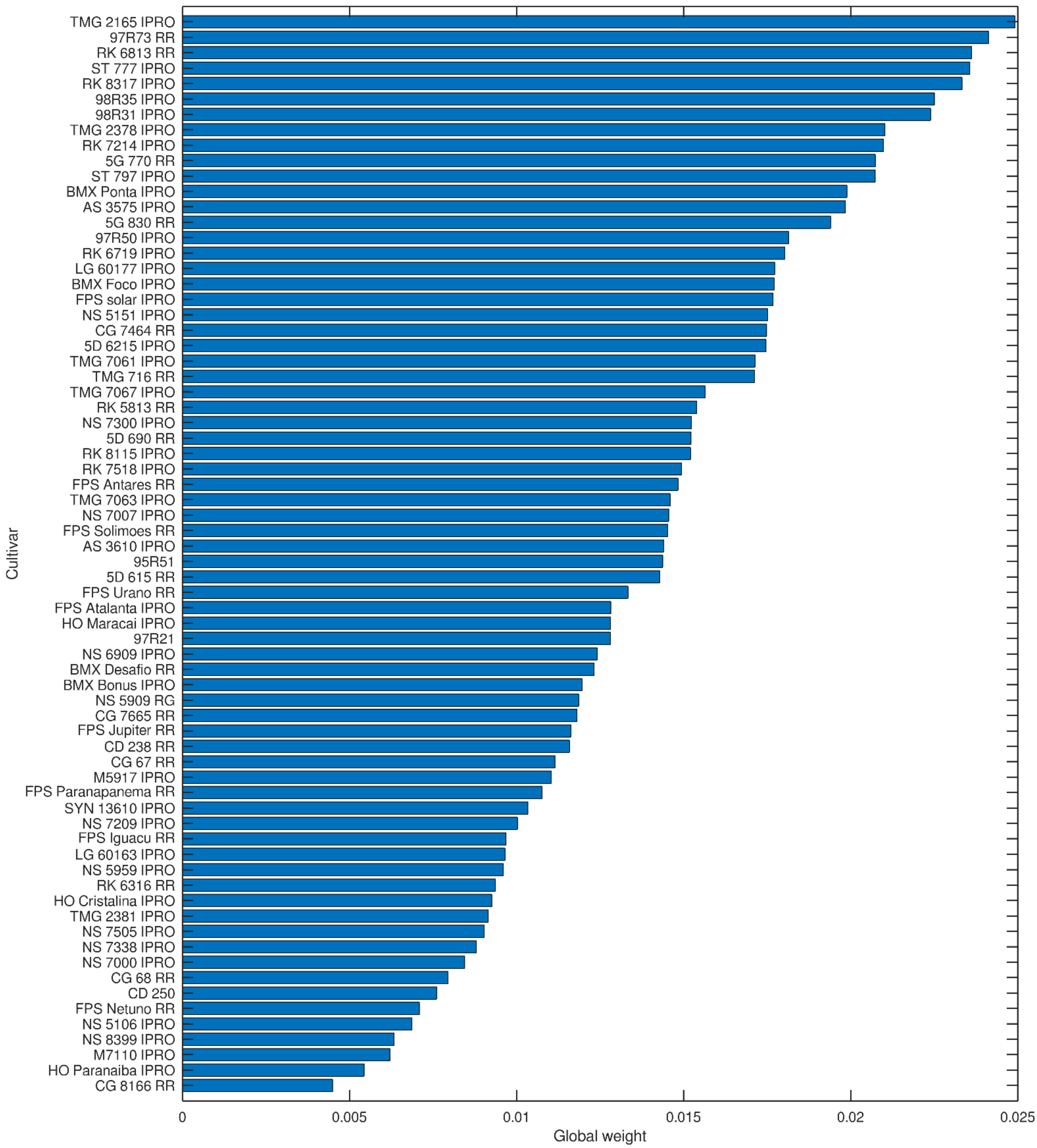

Figure 5. Result of the selection of soybean genotypes using the proposed approach.

Figure 5 presents the global weights obtained through the proposed approach. Cultivars are ranked in descending order, with higher weights signifying smaller distances across both stress environments compared to the control. Notably, in this specific research, weights were set equally at 0.5, deviating from the approach outlined in equation (9). This decision aligns with the original research (De Oliveira et al., 2022) where equal weights were employed.

The research by De Oliveira et al. (2024) utilized TOPSIS for selection and identified the following ten most-tolerant genotypes (ranked highest to lowest): RK 6813 RR, ST 777 IPRO, RK 7214 IPRO, TMG 2165 IPRO, 5G 830 RR, 98R35 IPRO, 98R31 IPRO, RK 8317 IPRO, CG 7464 RR, and LG 60177 IPRO. While the proposed approach generates a different ranking, there is significant agreement regarding specific cultivars within the top ten positions (RK 6813 RR, ST 777 IPRO, RK 7214 IPRO, TMG 2165 IPRO, 98R35 IPRO, 98R31 IPRO, and RK 8317 IPRO). However, the proposed method ranked cultivars 5G 830 RR (14th), LG 60177 IPRO (17th), and CG 7464 RR (21st) outside the top ten.

This discrepancy in ranking doesn't reflect a methodological flaw or improper application. It stems from the fundamental differences between the Analytic Hierarchy Process (AHP) and TOPSIS approaches (Jozaghi et al., 2018). AHP's reliance on the consistency ratio of the pairwise comparison matrix significantly impacts the final rankings (Supraja and Kousalya, 2016).

The second experiment employed the same methodology to select forage grasses suitable for limited water conditions. Nine grass varieties were assessed under two water stress levels: moderate and severe. This allowed for a more nuanced understanding of stress tolerance (De Oliveira et al., 2024).

Figure 6 illustrates the weights assigned by the proposed method for each forage grass variety. Similar to the soybean results, a comparison with the findings of De Oliveira et al. (2024) reveals some agreement in the selection order. Their research identified the following order based on the highest TOPSIS score: ADR 300, Pojuca, Marandu, Xaraes, Mombaca, BRS Piata, Comum, Aruana, and Tanzania. The sole discrepancy lies in the ranking of Xaraes and Mombaca varieties, which were reversed by the proposed method.

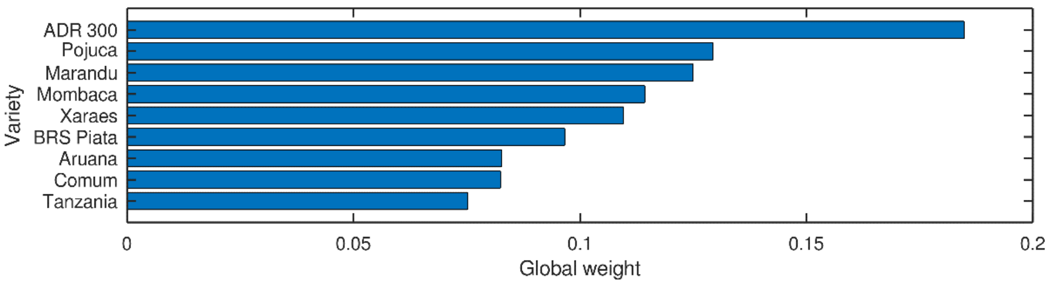

Figure 6. Result of the selection of forage grass variety using the proposed approach.

These applications demonstrate the viability of the proposed methodology for agricultural applications. Such applications hold immense relevance for food security and sustainability. In the face of climate changes, selecting cultivars resistant to water stress and other abiotic stresses is crucial. Additionally, it can lead to reduced reliance on fertilizers, promoting more sustainable agricultural practices. Furthermore, the proposed approach can incorporate the expertise of farmers, allowing them to factor in local environmental and soil conditions when selecting cultivars for their specific needs.

5. Conclusions

A new approach based on AHP was presented to reduce the effort required from decision-makers when making decisions based on tabular data. Both proposed strategies, automated and semi-automated, demonstrated consistency with state-of-the-art methods. A key advantage of this new approach is the ability to generate PC matrices, allowing the decision-maker's subjectivity and expertise to be applied only in the comparisons that require their knowledge. Future work will explore this approach in larger databases (big data) and seek a mechanism in the automated approach to prevent the generation of inconsistent matrices.

6. References

Alelaiwi, A. (2019). Evaluating distributed IoT databases for edge/cloud platforms using the analytic hierarchy process. Journal of Parallel and Distributed Computing, 124, 41-46. https://doi.org/10.1016/j.jpdc.2018.10.008

Aguarón, J., Escobar, M. T., Moreno-Jiménez, J. M., & Turón, A. (2019). AHP-group decision-making based on consistency. Mathematics, 7(3), 242. https://doi.org/10.3390/math7030242

Alonso, J. A., & Lamata, M. T. (2006). Consistency in the analytic hierarchy process: a new approach. International journal of uncertainty, fuzziness and knowledge-based systems, 14(04), 445-459. https://doi.org/10.1142/S0218488506004114

Bulut, E., & Duru, O. (2018). Analytic Hierarchy Process (AHP) in maritime logistics: theory, application and fuzzy set integration. In Multi-Criteria Decision-making in Maritime Studies and Logistics (pp. 31-78). Springer, Cham.

Costa, C. A. B. e, & Vansnick, J. C. (2008). A critical analysis of the eigenvalue method used to derive priorities in AHP. European Journal of Operational Research, 187(3), 1422-1428. http://dx.doi.org/10.1016/j.ejor.2006.09.022

Costa, J. F. S., Borges, A. R., & dos Santos Machado, T. (2016). Analytic Hierarchy Process applied to industrial location: A Brazilian perspective on jeans manufacturing. International Journal of the Analytic Hierarchy Process, 8(1), 77-91. http://dx.doi.org/10.13033/ijahp.v8i1.210

El Hefnawy, A., & Mohammed, A. S. (2014). Review of different methods for deriving weights in the Analytic Hierarchy Process. International Journal of the Analytic Hierarchy Process, 6(1), 92-123. http://dx.doi.org/10.13033/ijahp.v6i1.226

Franek, J., & Kresta, A. (2014). Judgment scales and consistency measure in AHP. Procedia Economics and Finance 12, 164-173. https://doi.org/10.1016/S2212-5671(14)00332-3

Ho, W., & Ma, X. (2018). The state-of-the-art integrations and applications of the analytic hierarchy process. European Journal of Operational Research, 267(2), 399-414. https://doi.org/10.1016/j.ejor.2017.09.007

Hutchinson, J. W., Alba, J. W., & Eisenstein, E. M. (2010). Heuristics and biases in data-based decision-making: Effects of experience, training, and graphical data displays. Journal of Marketing Research, 47(4), 627-642. https://doi.org/10.1509/jmkr.47.4.627

Jozaghi, A., Alizadeh, B., Hatami, M., Flood, I., Khorrami, M., Khodaei, N., & Ghasemi Tousi, E. (2018). A comparative study of the AHP and TOPSIS techniques for dam site selection using GIS: A case study of Sistan and Baluchestan Province, Iran. Geosciences, 8(12), 494. https://doi.org/10.3390/geosciences8120494

Krejčí, J., & Stoklasa, J. (2018). Aggregation in the analytic hierarchy process: Why weighted geometric mean should be used instead of weighted arithmetic mean. Expert Systems with Applications, 114, 97-106. https://doi.org/10.1016/j.eswa.2018.06.060

Khan, A. U., & Ali, Y. (2020). Analytical Hierarchy Process (AHP) and Analytic Network Process methods and their applications: a twenty year review from 2000-2019. International Journal of the Analytic Hierarchy Process, 12(3). 369-459. https://doi.org/10.13033/ijahp.v12i3.822

Koca, G., & Yıldırım, S. (2021). Bibliometric analysis of DEMATEL method. Decision-making: Applications in Management and Engineering, 4(1), 85-103. https://doi.org/10.31181/dmame2104085g

Kumar, R., Padhi, S. S., & Sarkar, A. (2019). Supplier selection of an Indian heavy locomotive manufacturer: An integrated approach using Taguchi loss function, TOPSIS, and AHP. IIMB Management Review, 31(1), 78-90. https://doi.org/10.1016/j.iimb.2018.08.008

Lamata, M. T., & Peláez, J. I. (2002). A method for improving the consistency of judgements. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 10(06), 677-686.

Leal, J. E. (2020). AHP-express: A simplified version of the analytical hierarchy process method. MethodsX, 7, 100748. https://doi.org/10.1016/j.mex.2019.11.021

Lin, C., & Kou, G. (2021). A heuristic method to rank the alternatives in the AHP synthesis. Applied Soft Computing, 100, 106916. https://doi.org/10.3390/math7030242

Melo, F. J. C. de, Sousa, J. V., de Aquino, J. T., & de Barros Jerônimo, T. (2021). Using AHP to improve manufacturing processes in TPM on industrial and port complex. Exacta, 19(3), 523-549. https://doi.org/10.5585/exactaep.2021.16693

Moeinaddini, M., Khorasani, N., Danehkar, A., & Darvishsefat, A. A. (2010). Siting MSW landfill using weighted linear combination and analytical hierarchy process (AHP) methodology in GIS environment (case study: Karaj). Waste management, 30(5), 912-920. https://doi.org/10.1016/j.wasman.2010.01.015

Nabeeh, N. A., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Neutrosophic multi-criteria decision-making approach for IoT-based enterprises. IEEE Access, 7, 59559-59574. https://doi.org/10.1109/ACCESS.2019.2908919

Oliveira, B. R. D., Oliveira, L. R., & Duarte, M. A. Q. (2016). Multicriteria Analysis Applied at the Choice of Projects Specified by Resolution nº 154/2012 of the National Council of Justice (Análise multicritério aplicada a escolha de projetos especificados pela resolução nº 154/2012 do conselho nacional de justiça). Democracia Digital e Governo Eletrônico, Florianópolis, 14, 121-142.

Oliveira, B. R. D., de Abreu, C. C. E., Duarte, M. A. Q., & Vieira Filho, J. (2019). Geometrical features for premature ventricular contraction recognition with analytic hierarchy process based machine learning algorithms selection. Computer methods and programs in biomedicine, 169, 59-69. https://doi.org/10.1016/j.cmpb.2018.12.028

Oliveira, B. R. D., Duarte, M. A. Q., & Vieira Filho, J. (2022). Premature Ventricular Contraction Recognition using Blind Source Separation and Ensemble Gaussian Naive Bayes weighted by Analytic Hierarchy Process. Acta Scientiarum. Technology, 44, e60386, 1-13. https://doi.org/10.4025/actascitechnol.v44i1.60386

de Oliveira, B. R., Zuffo, A. M., Aguilera, J. G., Steiner, F., Ancca, S. M., Flores, L. A. P., & Gonzales, H. H. S. (2022). Selection of soybean genotypes under drought and saline stress conditions using Manhattan distance and TOPSIS. Plants, 11(21), 2827. https://doi.org/10.3390/plants11212827

de Oliveira, B. R., Queiroz Duarte, M. A., Zuffo, A. M., Steiner, F., González Aguilera, J., Filgueiras Dutra, A., ... & Caviedes Contreras, W. (2024). Selection of forage grasses for cultivation under water-limited conditions using Manhattan distance and TOPSIS. Plos one, 19(1), e0292076. https://doi.org/10.1371/journal.pone.0292076

Rigo, P. D., Rediske, G., Rosa, C. B., Gastaldo, N. G., Michels, L., Neuenfeldt Júnior, A. L., & Siluk, J. C. M. (2020). renewable energy problems: Exploring the methods to support the decision-making process. Sustainability, 12(23), 10195, 1-27. https://doi.org/10.3390/su122310195

Saaty, T. L. (1986). Axiomatic foundation of the analytic hierarchy process. Management science, 32(7), 841-855.

Saaty, T. L. (1990). How to make a decision: the analytic hierarchy process. European journal of operational research, 48(1), 9-26.

Saaty, T. L. (2003). Decision-making with the AHP: Why is the principal eigenvector necessary. European journal of operational research, 145(1), 85-91. https://doi.org/10.1016/S0377-2217(02)00227-8

Saaty, T. L. (2016). The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision-making. In Multiple criteria decision analysis (pp. 363-419). Springer, New York, NY.

Saaty, T. L. (2008). Relative measurement and its generalization in decision-making why pairwise comparisons are central in mathematics for the measurement of intangible factors the analytic hierarchy/network process. RACSAM-Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas, 102(2), 251-318. https://doi.org/10.1007/BF03191825

Saaty, T. L., & Vargas, L. G. (2012). The seven pillars of the analytic hierarchy process. In Models, methods, concepts & applications of the analytic hierarchy process (pp. 23-40). Springer, Boston, MA. https://doi.org/10.1007/978-1-4614-3597-6_2

Santos, M. dos, Araujo Costa, I. P. de, & Gomes, C. F. S. (2021). Multicriteria decision-making in the selection of warships: a new approach to the AHP method. International Journal of the Analytic Hierarchy Process, 13(1), 147-169. https://doi.org/10.13033/ijahp.v13i1.833

Sbai, N., Benabbou, L., & Berrado, A. (2020). An AHP Based Approach for Multi-echelon Inventory System Selection: Case of Distribution Systems. In 2020 5th International Conference on Logistics Operations Management (GOL) (pp. 1-8). IEEE. https://doi.org/10.1109/GOL49479.2020.9314711

Shao, M., Han, Z., Sun, J., Xiao, C., Zhang, S., & Zhao, Y. (2020). A review of multi-criteria decision-making applications for renewable energy site selection. Renewable Energy, 157, 377-403. https://doi.org/10.1016/j.renene.2020.04.137

Supraja, S., & Kousalya, P. (2016). A comparative study by AHP and TOPSIS for the selection of all round excellence award. In 2016 International Conference on Electrical, Electronics, and Optimization Techniques (ICEEOT) (pp. 314-319). IEEE. https://doi.org/10.1109/ICEEOT.2016.7755271

Terzi, E. (2019). Analytic hierarchy process (AHP) to solve complex decision problems. Southeast Europe Journal of Soft Computing, 8(1), 6-12. http://dx.doi.org/10.21533/scjournal.v8i1.168.g162

Yu, D., Kou, G., Xu, Z., & Shi, S. (2021). Analysis of collaboration evolution in AHP research: 1982–2018. International Journal of Information Technology & Decision-making (IJITDM), 20(01), 7-36. https://dx.doi.org/10.1142/S0219622020500406

6. Additional Information

6.1 Data availability statement

The computational implementation is available at https://github.com/brunobro/ahptd

6.2 Conflicts of Interest

We declare that there is no conflict of interest.